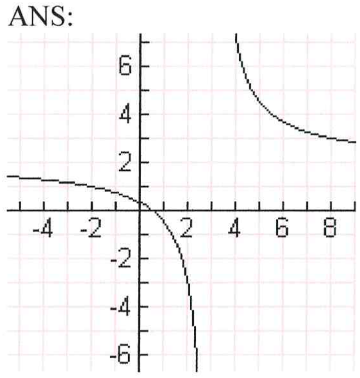
MHF4U Final Exam Review
Answer Section

SHORT ANSWER

1. ANS:
The degree of the numerator must be exactly one more than the degree of the denominator.
2. ANS:
The zeros are $x = 9$ and $x = -2$.
3. ANS:
The vertical asymptotes are $x = 0$ and $x = 5$.
4. ANS:
The domain is $\{x \mid x \neq 1, x \in R\}$.
5. ANS:
The x-intercepts are $x = 0, 3, -5$.
6. ANS:
The y-intercept is $\frac{1}{3}$.
7. ANS:


8. ANS:
There is a vertical asymptote at $x = 3$ and a horizontal asymptote at $y = 1$.
9. ANS:
The domain is $\{x \mid x \neq 1, -2, x \in R\}$. There is only one vertical asymptote at $x = -2$.
10. ANS:
In order to have vertical asymptotes of $x = 5$ and $x = -3$, the denominator must have the factors $(x-5)$ and $(x+3)$. In order to have a horizontal asymptote at $y = 0$, the degree of the numerator must be less than the degree of the denominator. To ensure an x-intercept of 2, the numerator must have $(x-2)$ as a factor. These various factors give us the following rational function: $f(x) = \frac{x-2}{(x-5)(x+3)}$. This function simplifies to
$$f(x) = \frac{x-2}{x^2 - 2x - 15}.$$
11. ANS:
The function has a vertical asymptote at $x = -2$. There is no horizontal asymptote since the degree of the numerator is greater than the degree of the denominator. In order to find oblique asymptotes, long division is required.

$$\begin{array}{r} x + 1 \\ x+2 \overline{) x^2 + 3x - 2} \\ \underline{x^2 + 2x} \\ x - 2 \\ \underline{x + 2} \\ -4 \end{array}$$

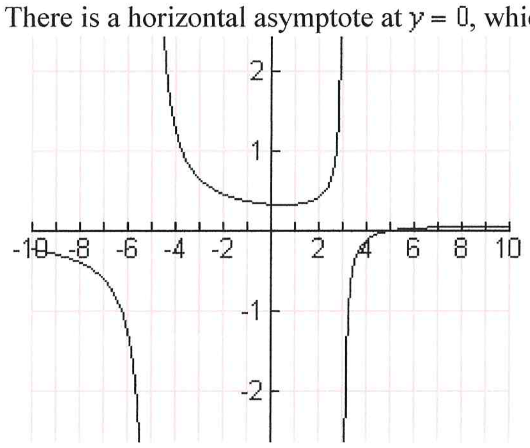
$$\therefore f(x) = \frac{x^2 + 3x - 2}{x + 2} \text{ is equal to } x + 1 - \frac{4}{x+2}.$$
Hence, the equation of the oblique asymptote is $y = x + 1$.

12. ANS:
Since there are vertical asymptotes at $x = -3$ and $x = 2$, the denominator of the function must include $(x+3)(x-2)$. This simplifies to $x^2 + x - 6$. So far, the function is $f(x) = \dfrac{k}{x^2 + x - 6}$. There appears to be a y-intercept at $y = -1$. Set x equal to zero while letting $f(x) = -1$. This gives a value of 6 for k. The equation is
$f(x) = \dfrac{6}{x^2 + x - 6}$.

13. ANS:
There is no oblique asymptote. There is a horizontal asymptote at $y = 1$, which can be found by creating tables. There is only one vertical asymptote at $x = 1$. When the function is factored into $f(x) = \dfrac{(x-2)(x+2)}{(x-1)(x-2)}$ and then simplified, the only remaining factor in the denominator is $x - 1$, giving only one vertical asymptote.

14. ANS:
In order to have restrictions on x, we must have the factors $(x+2)(x-7)$ in the denominator, but since there is no asymptote when $x = -2$, we should have $x + 2$ in the numerator as well. The function would then be
$f(x) = \dfrac{(x+2)}{(x-7)(x+2)}$. This function would look very similar to $f(x) = \dfrac{1}{x-7}$ but there would be a hole in the graph where $x = -2$.

15. ANS:
$f(x) = \dfrac{x-5}{(x+5)(x-3)}$
The domain is $\{x \mid x \neq -5, 3, x \in R\}$. The x-intercept of $x = 5$ is found by setting $f(x)$ equal to zero. The y-intercept of $y = \frac{1}{3}$ is found by setting x equal to zero. There is a vertical asymptote at $x = 3$ and $x = -5$.
There is a horizontal asymptote at $y = 0$, which can be found by creating tables.



16. ANS:
$f(x)$ increases for $x \in R$; $g(x)$ increases for $x < 3$; and $h(x)$ increases for $x < -1$ and $x > 2.5$.

17. ANS:

$f(x)$ – a); $g(x)$ – d); $h(x)$ – e)

18. ANS:

A local maximum is a point at which a curve changes from increasing to decreasing, whereas the local maximum value is the y-coordinate of that point.

19. ANS:

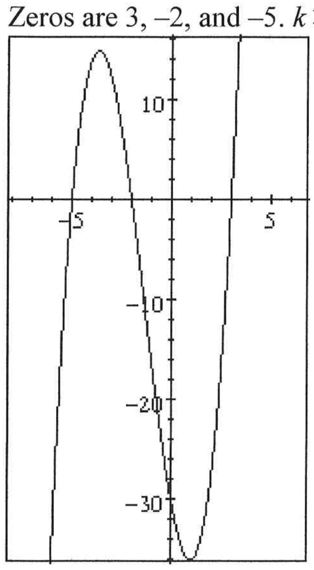
When a function changes from decreasing to increasing, a local minimum occurs.

20. ANS:

$f(x) = -2x^3 - 2x^2 + 10x - 6$

21. ANS:

Zeros are 3, –2, and –5. $k > 0 \Rightarrow$ as $x \to \infty, f(x) \to \infty$



22. ANS:
$r = 9$
23. ANS:

a) $x + 1$ is the divisor $\Rightarrow k = -1$

$$\begin{array}{ccccccl}
1x^4 & + 1x^3 & + 2x^2 & + 0x & + 1 & & \\
\downarrow & \downarrow & \downarrow & \downarrow & \downarrow & & \text{(remove literal coefficients)} \\
1 & 1 & 2 & 0 & 1 & & \\
\downarrow & \underline{-1} & \underline{0} & \underline{-2} & \underline{2} & & \text{(multiply number at lower left by } k\text{)} \\
1 & 0 & 2 & -2 & 3 & & \text{(add)} \\
\downarrow & \downarrow & \downarrow & \downarrow & \downarrow & & \text{(insert literal coefficients)} \\
\end{array}$$

$1x^3 + 0x^2 + 2x - 2$ remainder 3

b) $(x + 1)(1x^3 + 2x - 2) + 3 = x^4 + x^3 + 2x^2 + 1$

24. ANS:
Yes
25. ANS:
The remainder is 13.
26. ANS:

$$2x + 1 \overline{\big) 2x^3 + 5x^2 - 4x - 5}^{\;x^2 + 2x - 3} \text{ remainder } -2$$

27. ANS:

$2x + 1$ is the divisor $\Rightarrow k = -\dfrac{1}{2}$

$$\begin{array}{ccccl}
2x^3 & + 5x^2 & - 4x & - 5 & \\
\downarrow & \downarrow & \downarrow & \downarrow & \text{(remove literal coefficient)} \\
2 & 5 & -4 & -5 & \\
\downarrow & \underline{-1} & \underline{-2} & \underline{3} & \text{(multiply number at lower left by } k\text{)} \\
2 & 4 & 6 & -2 & \text{(add)} \\
\downarrow & \downarrow & \downarrow & \downarrow & (\div\, 2\text{, coefficient of } x \text{ in divisor)} \\
1 & 2 & -3 & \downarrow & \\
\downarrow & \downarrow & \downarrow & \downarrow & \text{(insert literal coefficients)} \\
\end{array}$$

$1x^2 + 2x - 3$ remainder –2

28. ANS:

$x^3 - 5x^2 - x + 5 = (x - 1)(x - 5)(x + 1)$

29. ANS:

$4x^3 + 4x^2 - x - 1 = (2x - 1)(2x + 1)(x + 1)$

30. ANS:

$x^3 - 1000 = (x - 10)(x^2 + 10x + 100)$

31. ANS:

$x^4 - 7x^2 - 6x = x(x + 1)(x - 3)(x + 2)$

32. ANS:
$x = -1, 1, 3$

33. ANS:

$x(2x + 1)(x - 4) > 0$

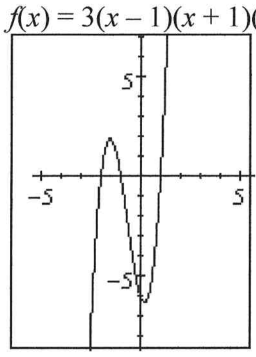
The zeros of the corresponding equation are $-\frac{1}{2}$, 0, and 4.

	sign of x	sign of $(2x + 1)$	sign of $(x - 4)$	sign of product
$x < -\frac{1}{2}$	–	–	–	–
$-\frac{1}{2} < x < 0$	–	+	–	+
$0 < x < 4$	+	+	–	–
$x > 4$	+	+	+	+

$x(2x + 1)(x - 4) > 0$ when $-\frac{1}{2} < x < 0$ and $x > 4$.

34. ANS:

The given zeros: $f(x) = k(x - 1)(x + 1)(x + 2)$
y-intercept: $f(0) = -6$
$f(0) = k(0 - 1)(0 + 1)(0 + 2)$
$-6 = k(-2)$
$3 = k$
$f(x) = 3(x - 1)(x + 1)(x + 2)$ is the required function.



35. ANS:

a) When $f(x)$ is divided by $2x - 1$, the remainder $= f\left(\frac{1}{2}\right)$

$f(x) = 4x^4 - x^3 + 2x^2 - 1$
$$f\left(\frac{1}{2}\right) = 4\left(\frac{1}{16}\right) - \left(\frac{1}{8}\right) + 2\left(\frac{1}{4}\right) - 1 = \frac{1}{4} - \frac{1}{8} + \frac{1}{2} - 1 = -\frac{3}{8}$$

Remainder $= -\frac{3}{8}$

b) If $2x - 1$ is a factor of $4x^4 - x^3 + 2x^2 - 1$, then $f\left(\frac{1}{2}\right) = 0$.

However, $f\left(\frac{1}{2}\right) = -\frac{3}{8}$, so $2x - 1$ is not a factor of $4x^4 - x^3 + 2x^2 - 1$.

36. ANS:

$f(x) = x^3 - 3x^2 + 5x - 3$
$f(1) = 1 - 3 + 5 - 3 = 0 \Rightarrow x - 1$ is a factor

$$x - 1 \overline{\smash{\big)}\, x^3 - 3x^2 + 5x - 3} \quad \text{quotient } x^2 - 2x + 3$$ remainder 0

$(x - 1)(x^2 - 2x + 3) = 0$
$x - 1 = 0$ or $x^2 - 2x + 3 = 0$

$$x = 1 \text{ or } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{2 \pm \sqrt{(-2)^2 - 4(1)(3)}}{2(1)}$$

$$= \frac{2 \pm \sqrt{-8}}{2}$$

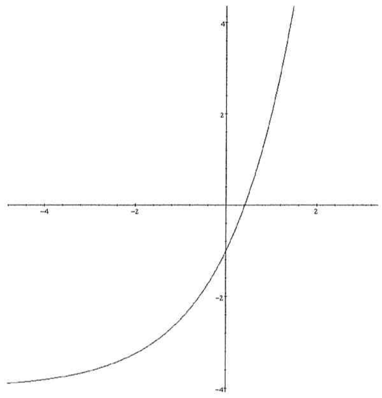
$$= \frac{2 \pm 2i\sqrt{2}}{2}$$

$$= 1 \pm i\sqrt{2}$$

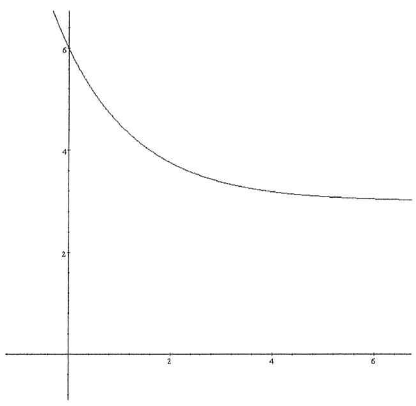
$x = 1, 1 + i\sqrt{2}$

37. ANS:
8.73 g.
38. ANS:
y = -2
39. ANS:
(0 , -1)
40. ANS:
A vertical translation 4 units down.
41. ANS:
A vertical stretch by a factor of 3.
42. ANS:
x-intercept @ x = 1. There is no y-intercepts.
43. ANS:
A reflection in the line y = x.
44. ANS:
They are inverses of each other.
45. ANS:
5
46. ANS:
-3
47. ANS:
1
48. ANS:
4
49. ANS:
$\log_b\left(\frac{128}{9}\right)$
50. ANS:
5
51. ANS:
x = 4.86
52. ANS:
x = 3.30
53. ANS:
x = 1.63

54. ANS:
Domain: $x \in R$, Range: $y > -4, y \in R$, Asymptote: $y = -4$, y-intercept: $(0, -1)$



55. ANS:
Domain: $x \in R$, Range: $y > 1, y \in R$, Asymptote: $y = 3$, y-intercept: $(0, 6)$

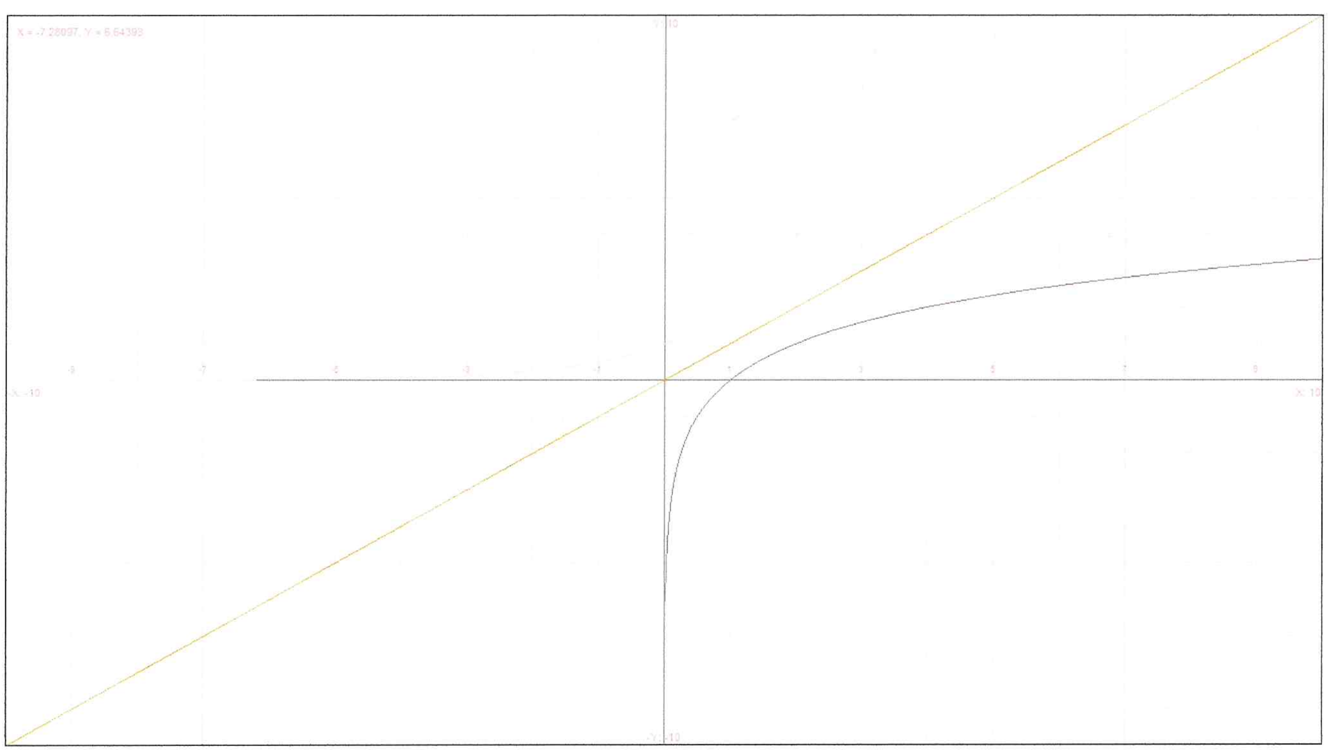


56. ANS:

Domain: $x \in R$ Range: $y > 7$, y-intercept (when $x = 0$): $y = 2(3)^0 + 7 = 9$, asymptote: $y = 7$

57. ANS:

As shown below, the graph of $y = \log_2 x$ is a reflection of the graph of $y = 2^x$ in the line $y = x$



58. ANS:

$$\log_8 6 - \log_8 3 + \log_8 4 = \log_8\left(\frac{6 \times 4}{3}\right) = \log_8 8 = 1$$

59. ANS:

$$\log_4(x-1)-\log_4(x+2)=1 \rightarrow \log_4\left(\frac{x-1}{x+2}\right)=\log_4 4$$

$$\frac{x-1}{x+2}=4 \rightarrow x-1=4x+8 \rightarrow 3x=-9 \rightarrow x=-3$$

Since $x > 1$, there is no solution.

60. ANS:

$$\log_3 x+\log_3 3=\log_3 1+\log_3 4 \rightarrow \log_3 x=\log_3\left(\frac{1\times 4}{3}\right) \rightarrow x=\frac{4}{3}$$

61. ANS:

$$\mathrm{pH}=-\log[\mathrm{H}^+] \rightarrow 2=-\log[\mathrm{H}_1^+] \rightarrow \mathrm{H}_1^+=10^{-2}$$

$$8=-\log[\mathrm{H}_2^+] \rightarrow \mathrm{H}_2^+=10^{-8}$$

Change factor: $\dfrac{\mathrm{H}_2^+}{\mathrm{H}_1^+}=\dfrac{10^{-8}}{10^{-2}}=10^{-6}$

62. ANS:

$$M=c(2)^{\frac{t}{D}} \rightarrow 1241=134(2)^{\frac{24}{D}} \rightarrow \frac{1241}{134}=2^{\frac{24}{D}} \rightarrow \frac{24}{D}=\frac{\log\left(\frac{1241}{134}\right)}{\log 2}$$

$$D=\frac{24\log 2}{\log(\frac{1241}{134})}=7.47$$

The doubling period is 7.5 hours.

63. ANS:
48

64. ANS:
3a + b

65. ANS:
-9.8

66. ANS:
-24.8

67. ANS:
0 m/s

68. ANS:
2 m/s

69. ANS:

The particle is moving backwards at $\frac{17}{361}$ cm/s.

70. ANS:
a)

Interval	Δd	Δt	**Average Rate of Change,** $\frac{\Delta d}{\Delta t}$
$5 \le t \le 6$	53.9	1	53.9
$5.5 \le t \le 6$	28.175	0.5	56.35
$5.9 \le t \le 6$	5.831	0.1	58.31
$5.99 \le t \le 6$	0.58751	0.01	58.751

b) 58.75 m/s

71. ANS:
Domain is {1, 2, 5}.

72. ANS:
Domain = {–2, 2}.

73. ANS:
$h(2) = 225$

74. ANS:
$g(f(2)) = -37$

75. ANS:
$g \circ g^{-1} = \{(1, 1), (2, 2), (3, 3)\}$ and $f \circ g = \{(2, 3), (4, 5)\}$

76. ANS:
If $f(x) = 2x + 3$, then $y = 2x + 3$ has the inverse $x = 2y + 3$ or $y = \dfrac{x-3}{2}$.

Therefore, $f^{-1}(x) = \dfrac{x-3}{2}$.

$$\left(f \circ f^{-1}\right)(x) = f\left(f^{-1}(x)\right)$$
$$= 2\left(\frac{x-3}{2}\right) + 3$$
$$= x - 3 + 3$$
$$= x$$

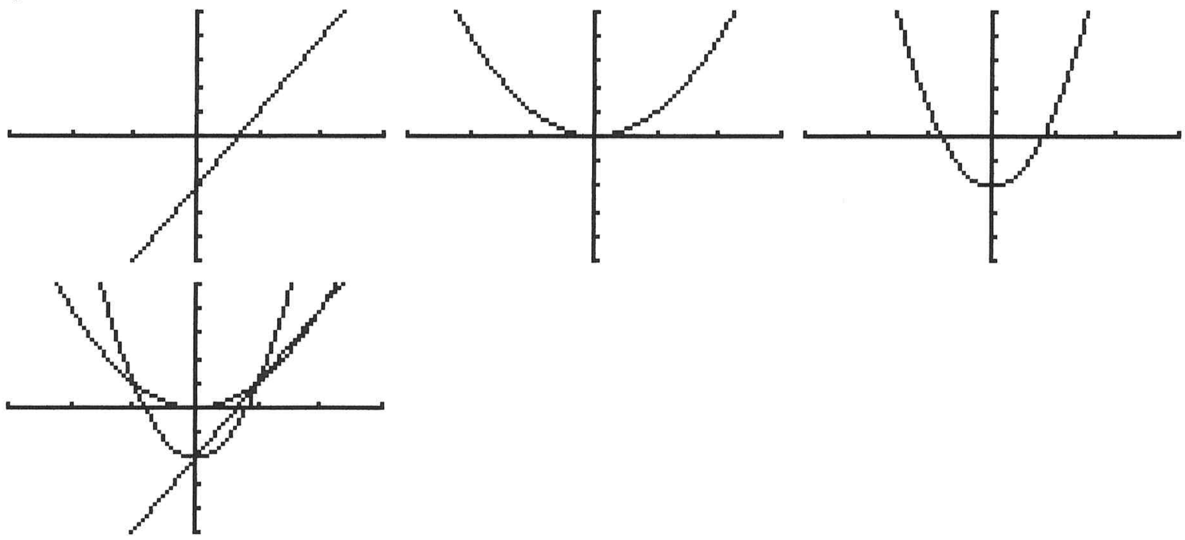
77. ANS:
Let $g(x) = v$.
$f(g(x)) = f(v) = v^2 + 3v$
But $f(g(x)) = x^2 + 7x + 10$, so $v^2 + 3v = x^2 + 7x + 10$.
R.S. $= (x + 5)(x + 2)$
$= (x + 2 + 3)(x + 2)$
Let $v = x + 2$.
Then R.S. $= (v + 3)v =$ L.S.
Therefore, $g(x) = x + 2$.

78. ANS:
a) Let the function $B(h)$ represent Bill's hourly earnings for one day. Then $B(h) = 30h + 23$.
Then let the function $U(B(h))$ represent the union dues paid for each day's earnings. Therefore, $U(B(h)) = 0.015(30h + 23)$ or $U(B(h)) = 0.45h + 0.345$.
b)

Hours Worked	$(U \circ B)(h)$ Daily Earnings	Union Dues
h	$B(h)$	$U(B(h))$
1	53	0.795
2	83	1.245
3	113	1.695
…	…	…
8	263	3.945
Domain of B	Range of B Domain of U	Range of U

c) $U(B(h)) = 0.45h + 0.345$
$U(B(8)) = 0.45(8) + 0.345 = 3.945$
The union dues are \$3.95/d.

79. ANS:
a) $\left(f\circ g\right)(x) = f(g(x)) = 3(x^2) - 2 = 3x^2 - 2$
b)



c) The parabola $y = x^2$ has a vertical stretch of 3 units and a vertical translation of –2 units.

80. ANS:

There will be 2 solutions that will occur in Quadrants 3 and 4. The related acute angle will be $\frac{\pi}{6}$. Therefore the solutions are $x = \frac{7\pi}{6}$ *and* $\frac{11\pi}{6}$.

81. ANS:
a) $\frac{7\pi}{6}$ b) $\frac{3\pi}{4}$ c) 0.57 d) 36^0 e) 330^0 f) 45.86^0

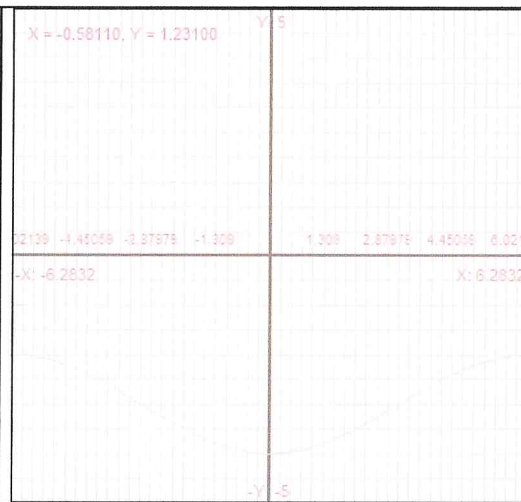
82. ANS:
a) $-\sqrt{3}$ b) $-\frac{\sqrt{3}}{2}$ c) $\frac{1}{2}$ d) $-\frac{1}{2}$

83. ANS:

a) $D{:}\{x \in \Re\}$

$R{:}\left\{y \in \Re \mid -4 \le y \le -2\right\}$

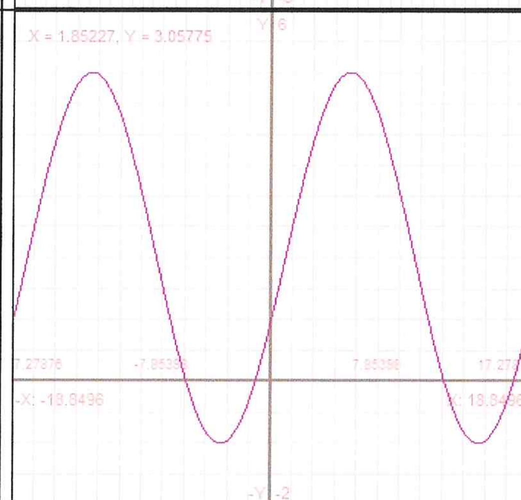
Amplitude = 1
Period = 4π
Phase Shift = π right



b) $D{:}\{x \in \Re\}$

$R{:}\left\{y \in \Re \mid -1 \le y \le 5\right\}$

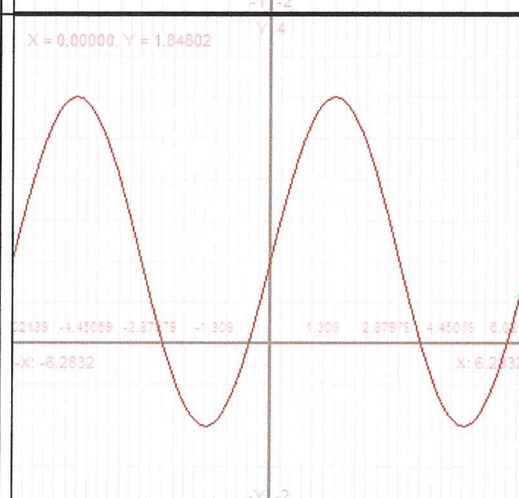
Amplitude = 3
Period = 6π
Phase Shift = $\dfrac{\pi}{3}$ right



c) $D{:}\{x \in \Re\}$

$R{:}\left\{y \in \Re \mid -1 \le y \le 3\right\}$

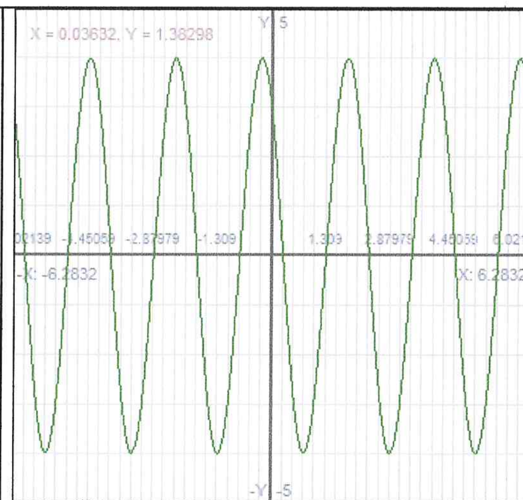
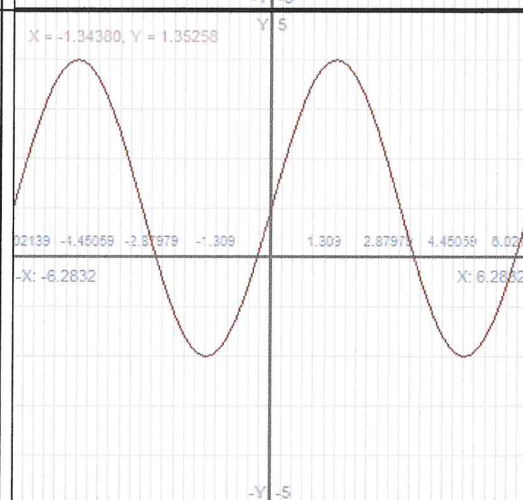
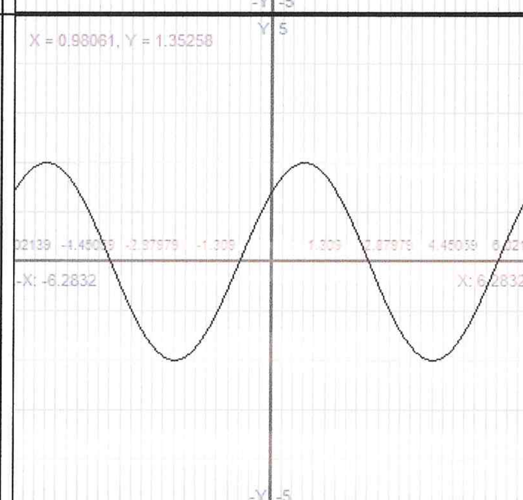
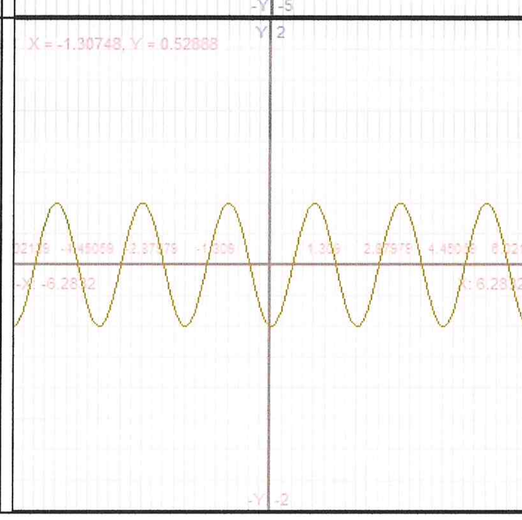
Amplitude = 2
Period = 2π
Phase Shift = N/A



d) $D{:}\{x \in \Re\}$

$R{:}\left\{y \in \Re \mid -4 \le y \le -2\right\}$

Amplitude = 1
Period = 4π
Phase Shift = π right

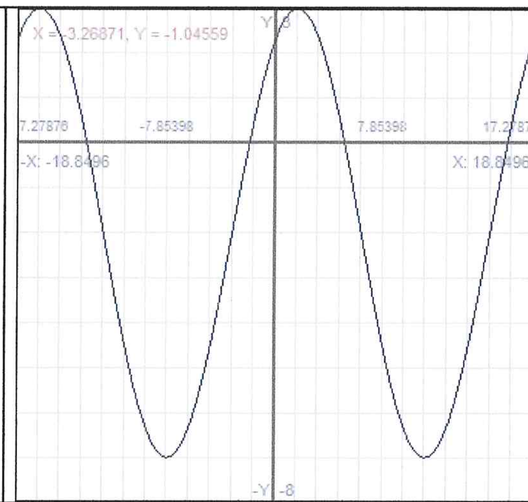
e) $D:\{x \in \Re\}$ $R:\{y \in \Re \mid -4 \le y \le 4\}$ Amplitude = 4 Period = $\dfrac{2\pi}{3}$ Phase Shift = $\dfrac{\pi}{4}$ right	
f) $D:\{x \in \Re\}$ $R:\{y \in \Re \mid -2 \le y \le 4\}$ Amplitude = 3 Period = 2π Phase Shift = N/A	
g) $D:\{x \in \Re\}$ $R:\{y \in \Re \mid -2 \le y \le 2\}$ Amplitude = 2 Period = 2π Phase Shift = $\dfrac{\pi}{4}$ right	
h) $D:\{x \in \Re\}$ $R:\{y \in \Re \mid -0.5 \le y \le 0.5\}$ Amplitude = 0.5 Period = $\dfrac{2\pi}{3}$ Phase Shift = $\dfrac{\pi}{2}$ left	

i) $D{:}\{x \in \Re\}$

$R{:}\left\{y \in \Re \mid -7 \le y \le 3\right\}$

Amplitude = 5
Period = 6π

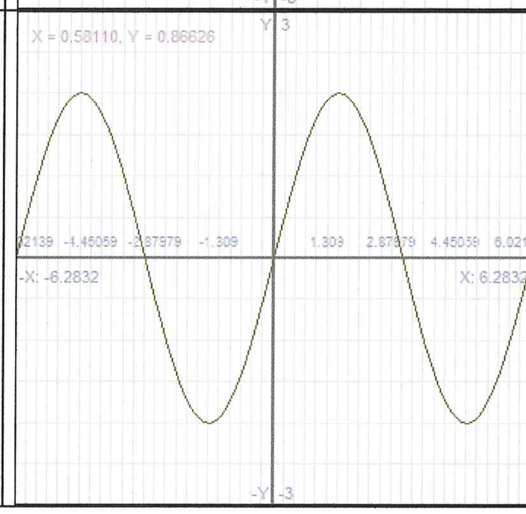
Phase Shift = $\dfrac{\pi}{2}$ right



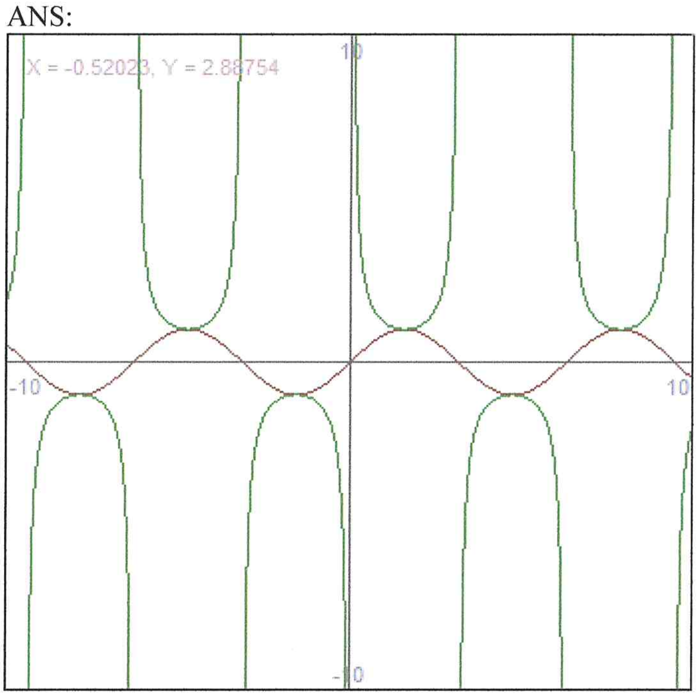
j) $D{:}\{x \in \Re\}$

$R{:}\left\{y \in \Re \mid -2 \le y \le 2\right\}$

Amplitude = 2
Period = 2π
Phase Shift = π right



84. ANS:



b)

Function	$y = \sin x$	$y = \csc x$
Domain	$\{x \in \Re\}$	$\left\{x \in \Re \mid x \ne \pm n\pi\right\}$
Range	$\left\{y \in \Re \mid -1 \le y \le 1\right\}$	$\left\{y \in \Re \mid y \ge 1 \text{ or } y \le -1\right\}$
Period	2π	2π
Key Points	$\left\{(0,0), \left(\frac{\pi}{2}, 1\right), (\pi, 0), \left(\frac{3\pi}{2}, -1\right), (2\pi, 0)\right\}$	$\text{minimum } @\ x = \frac{\pi}{2} \pm n(2\pi)$ $\text{maximum } @\ x = \frac{3\pi}{2} \pm n(2\pi)$
Asymptotes	None	$x = \pm n\pi$

85. ANS:

a) $\sin x(2+\cos x)=0$

$\therefore \sin x=0 \ and \ \cos x=-2$

$x=0,\pi,2\pi \ and \ N.P.$

b) $2\tan^2 x+3\tan x+1=0$

$(\tan x+1)(2\tan x+1)=0$

$\therefore \tan x=-1 \ and \ \tan x=-\frac{1}{2}$

$x=\frac{3\pi}{4},\frac{7\pi}{4} \ and \ x=2.68,5.82$

c) $4\sin x\cos x-2\sqrt{3}\sin x+2\cos x-\sqrt{3}=0$

$2\sin x\left(2\cos x-\sqrt{3}\right)+1\left(2\cos x-\sqrt{3}\right)=0$

$(2\sin x+1)\left(2\cos x-\sqrt{3}\right)=0$

$\therefore \sin x=\frac{-1}{2} \ and \ \cos x=\frac{\sqrt{3}}{2}$

$x=\frac{7\pi}{6},\frac{11\pi}{6} \ and \ x=\frac{5\pi}{6}$

d) $2+\cos x=2\cos x+3$

$\cos x=-1$

$\therefore x=\pi$

e) $2\cos x+\sin x\cos x=0$

$\cos x(2+\sin x)=0$

$\therefore \cos x=0 \ and \ \sin x=-2$

$x=\frac{\pi}{2},\frac{3\pi}{2} \ and \ N.P.$

f) $3\sin^2 x+7\sin x+2=0$

$(3\sin x+1)(\sin x+2)=0$

$\therefore \sin x=\frac{-1}{3} \ and \ \sin x=-2$

$x=3.48\ ,5.94 \ and \ N.P.$

g) $2\cos\vartheta-\sqrt{3}=0$

$\cos\vartheta=\frac{\sqrt{3}}{2}$

$\therefore \vartheta=\frac{\pi}{6},\frac{11\pi}{6}$

h) $7\sin\vartheta=5\sin\vartheta-1$

$2\sin\vartheta=-1$

$\therefore \sin\vartheta=\frac{-1}{2}$

$\vartheta=\frac{7\pi}{6},\frac{11\pi}{6}$

i) $4\sin^2 x+2\sin x-2=0$

$2\left(2\sin^2 x+\sin x-1\right)=0$

$2(2\sin x-1)(\sin x+1)=0$

$\therefore \sin x=\frac{1}{2} \ and \ \sin x=-1$

$x=\frac{\pi}{6},\frac{5\pi}{6} \ and \ x=\frac{3\pi}{2}$

j) $5\sin^2 x-18\sin x-8=0$

$(\sin x-4)(5\sin x+2)=0$

$\therefore \sin x=4 \ and \ \sin x=\frac{-2}{5}$

$N.P. \quad and \ \ x=3.55\ ,\ 5.87$

k) $4-4\cos x=4\sin^2 x-1$

$$4\sin^2 x+4\cos x-5=0$$

$$4\left(1-\cos^2 x\right)+4\cos x-5=0$$

$$4-4\cos^2 x+4\cos x-5=0$$

$$-4\cos^2 x+4\cos x-1=0$$

$$4\cos^2 x-4\cos x+1=0$$

$$(2\cos x-1)^2=0$$

$$\therefore\ \cos x=\frac{1}{2}$$

$$x=\frac{\pi}{3},\frac{5\pi}{3}$$

86. ANS:

a) LS: $\dfrac{1}{\cos x}+\tan x$

$=\dfrac{1}{\cos x}+\dfrac{\sin x}{\cos x}$

$=\dfrac{1+\sin x}{\cos x}$

RS: $\dfrac{\cos x}{1-\sin x}$

$=\dfrac{\cos x}{1-\sin x}\times\dfrac{1+\sin x}{1+\sin x}$

$=\dfrac{(\cos x)(1+\sin x)}{1-\sin^2 x}$

$=\dfrac{(\cos x)(1+\sin x)}{\cos^2 x}$

$=\dfrac{1+\sin x}{\cos x}$

b) LS: $1-2\tan x+\tan^2 x$

$=(\tan x-1)^2$

$=\left(\dfrac{\sin x}{\cos x}-1\right)^2$

$=\left(\dfrac{\sin x-\cos x}{\cos x}\right)^2$

$=\left(\dfrac{\sin^2 x-2\sin x\cos x+\cos^2 x}{\cos^2 x}\right)$

$=\dfrac{1-2\sin x\cos x}{\cos^2 x}$

RS: $\dfrac{1-2\sin x\cos x}{\cos^2 x}$

c) LS: $\dfrac{\cos^2 x}{1+3\sin x-4\sin^2 x}$

$=\dfrac{1-\sin^2 x}{-\left(4\sin^2 x-3\sin x-1\right)}$

$=\dfrac{(1-\sin x)(1+\sin x)}{-(\sin x-1)(4\sin x+1)}$

$=\dfrac{1+\sin x}{1+4\sin x}$

RS: $\dfrac{1+\sin x}{1+4\sin x}$

d) LS: $\dfrac{\cos x-\sin x}{\cos^2 x}$

RS: $\dfrac{1-\tan x}{\cos x}$

$=\dfrac{\left(1-\dfrac{\sin x}{\cos x}\right)}{\cos x}$

$=\left(\dfrac{\cos x-\sin x}{\cos x}\right)\div\cos x$

$=\dfrac{\cos x-\sin x}{\cos^2 x}$

e) LS: $\dfrac{\sin x}{\cos^2 x \tan x}$ $\quad$ RS: $\dfrac{1}{\cos x}$

$$= \frac{\sin x}{\cos^2 x\left(\dfrac{\sin x}{\cos x}\right)}$$

$$= \frac{\sin x}{\sin x \cos x}$$

$$= \frac{1}{\cos x}$$

f) LS: $\sin x$ $\quad$ RS: $\dfrac{1}{\sin x} - \dfrac{\cos x}{\tan x}$

$$= \frac{1}{\sin x} - \frac{\cos^2 x}{\sin x}$$

$$= \frac{\sin^2 x}{\sin x}$$

$$= \sin x$$

g) LS: $\dfrac{\sin^2 x + \cos^2 x}{1+\cos x} + \dfrac{1}{1-\cos x}$ $\quad$ RS: $\dfrac{2}{\sin^2 x}$

$$= \frac{1(1-\cos x) + 1(1+\cos x)}{(1+\cos x)(1-\cos x)}$$

$$= \frac{2}{1-\cos^2 x}$$

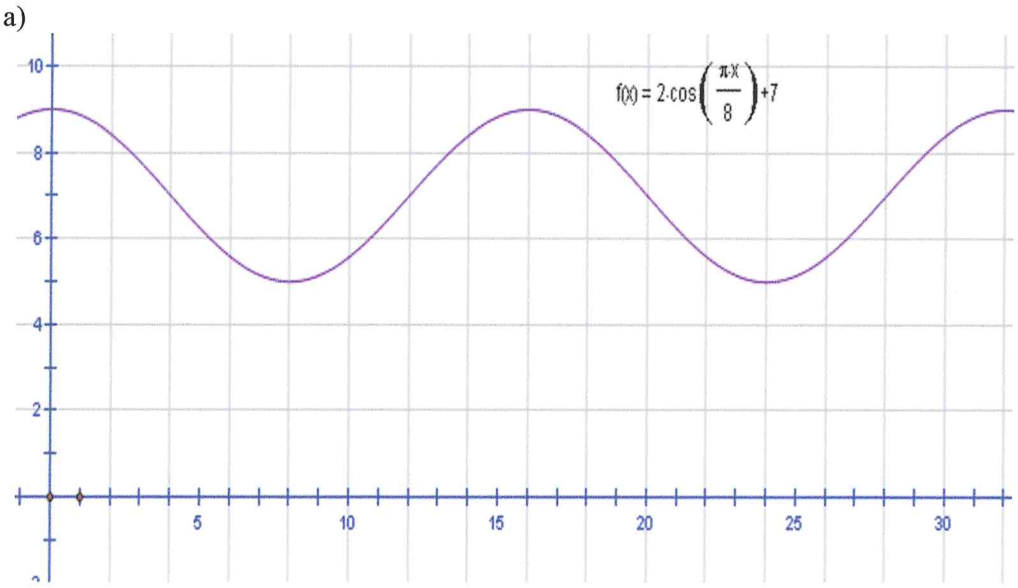
$$= \frac{2}{\sin^2 x}$$

h) LS: $\dfrac{\cos A - \sin 2A}{\cos 2A + \sin A - 1}$ $\quad$ RS: $\cot A$

$$= \frac{\cos A - 2\sin A\cos A}{\left(1-2\sin^2 A\right) + \sin A - 1} \qquad = \frac{\cos A}{\sin A}$$

$$= \frac{\cos A(1-2\sin A)}{\sin A(1-2\sin A)}$$

$$= \frac{\cos A}{\sin A}$$

87. ANS:
a)


b) period = $\frac{2\pi}{k}$

$16 = \frac{2\pi}{k}$

$k = \frac{\pi}{8}$

amplitude = $\frac{9-5}{2} = 2$

h(t)=2cos$\frac{\pi}{8}t + 7$

c) $\approx 5.6m$